Mathematical modeling is also physics—interdisciplinary teaching between mathematics and physics in Danish upper secondary education

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Mathematical modeling is also physics—interdisciplinary teaching between mathematics and physics in Danish upper secondary education

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Abstract

Mathematics plays a crucial role in physics. This role is brought about predominantly through the building, employment, and assessment of mathematical models, and teachers and educators should capture this relationship in the classroom in an effort to improve students' achievement and attitude in both physics and mathematics. But although there are overwhelming amounts of literature on modeling in science and mathematics education, the interdisciplinary position is seldom addressed explicitly. Furthermore, there has been a striking lack of exposure of the question of how future teachers, who are largely educated in a mono-disciplinary fashion, can best become equipped to introduce genuinely interdisciplinary teaching activities to their future pupils. This paper presents some preliminary reflections upon a graduate course, which aims to prepare future physics and mathematics teachers for interdisciplinary teaching, and which has been designed on the basis of influential theoretical expositions of the concept of interdisciplinarity.

Introduction

The Danish upper secondary education is organized in specialized subject packages containing compulsory subjects, core subjects, and elective subjects. An important feature of a package is that the core subjects form a coherent program, which is ensured by a closer interaction between the subjects. Some of the packages include the subjects of mathematics and physics. It is explicitly stated in the regulations from the ministry of education that if physics is part of a subject package including mathematics, then special teaching modules should be organized, in which the two disciplines work together on examining models of physical systems [1]. To fulfill the objective of coherence in the subject packages, interdisciplinary teaching is demanded. In an attempt to offer future science and mathematics teachers the possibility to prepare themselves for the practical challenges of interdisciplinary teaching, the University of Southern Denmark offer the graduate course 'Modeling and interdisciplinary teaching' for upper secondary education teacher-students. The course title explicitly highlights that the main idea of the course is modeling as a tool for interdisciplinary teaching.

This paper focuses on how to strengthen the educational relations between mathematics and physics by interdisciplinary teaching centered on modeling activities. As a background the interdisciplinary aspect of the scientific practice of mathematics and physics is discussed shortly. The main part of the paper consists of presentations of the didactical framework for interdisciplinary teaching applied in the course 'Modeling and interdisciplinary teaching' and series of illustrative examples of students' work taken from the implementation of the course.

Mathematics and physics

Most physicist and mathematicians are familiar with Eugene Wigner's famous article 'The unreasonable effectiveness of mathematics in the natural sciences'. Wigner describes the usefulness of mathematics in sciences as bordering on the mysterious, with accuracy beyond all reasonable expectations, and with no rational explanation for it [2]. Revisiting Wigner's theme, Lützen argues that extra-mathematical areas are central driving forces behind the development of mathematics. In particular, since the 17th century, physics has been an important field of inspiration for mathematics. The development of geometry and analysis has been shaped by physics from the beginning all the way up until the 20th century. Questions of the natural world have given rise to mathematical problems, and mathematical concepts; conjectures have been formulated with a view to intuitive physical situations and even solutions to problems and methods of proof have often been suggested by the behaviour of the physical world [3].

Wigner's theme is also challenged by some of the most prominent members of the international mathematical community. To celebrate the World Mathematical Year 2000 the American Mathematical Society published *Mathematics: Frontiers and Perspectives*, and in the books' preface the world famous mathematician Michael Atiyah reflects over mathematics in the 21st century and concludes that the close symbiotic relationship between mathematics and physics will rise to new heights in the 21st century [4]. The late Russian mathematician Vladimir Arnold characterizes mathematics as the part of physics where experiments are cheap, and marks the consequences of attempts to separate physics and mathematics as catastrophic, and warns that such attempts result in teaching ugly scholastic pseudo-mathematics to schoolchildren, where the scheme used in physics (observation, model, investigation of the model, conclusion, testing by observations) is replaced by the scheme definition, theorem, and proof [5].

In the context of upper secondary education it is worth noticing that several of the core concepts of mathematics and physics invite joint instructional sequences between the two subjects. Matthews *et al* point out that pendulums can be used to demonstrate a wide range of physical phenomena and provide a context in which students can become acquainted with mathematical modeling [6]. And one could add along with Osborne, that in science education it is often noted that many phenomena and their patterns of interaction are best described in the language of mathematics, which then becomes a bridge between the students' verbal language and the scientific meaning we seek to express [7].

A didactical framework for interdisciplinary teaching

Modeling is accepted as an important issue of mathematics and science education at all levels [8, 9], and this fact is the starting point of the course 'Modeling and interdisciplinary teaching'. The course is designed upon a framework with two pillars: (i) a didactical model for coordination and mutual interaction between mathematics and the subjects of natural science, and (ii) the conception of modeling as an interdisciplinary competency.

The didactical model applied in the course is inspired by the notion of 'emerging modeling'. The departing point of 'emerging modeling' is situation specific problems, which are subsequently modeled. The problems first offer the pupils the opportunity to develop situation-specific methods and symbolizations. Then the methods and symbols are modeled from a mathematical perspective, and in this sense mathematical models emerge from the pupils' activity. The models first come into being as a model of the situation, and then the model gradually becomes an entity in its own right and begins to serve as a model for mathematical reasoning. The shift presented from 'a model of' to 'a model for' should concur with a shift in the way pupils perceive and think about the models; from models that derive their meaning from the context situation modeled to thinking about the mathematical content of the models [10]. The didactical model applied in the course is an extension of the notion of 'emerging modeling' to include interdisciplinary activities between mathematics and subjects of natural sciences. The model consists of two phases: horizontal linking and vertical structuring. In the phase of horizontal linking, thematic integration is used to connect concept and process skills of mathematics and natural sciences by modeling activities in an interdisciplinary context, e.g. modeling the process of radioactive decay. The vertical structuring phase is characterized by a conceptual anchoring of the concepts, e.g. exponential growth and radioactivity, and process skills from the horizontal linking phase by creating languages and symbol systems that allow the pupils to move about logically and analytically within mathematics and the relevant subject(s) of natural sciences without reference back into the contextual phase. The shift from the horizontal linking to the vertical structuring phase might thus concur with a shift from interdisciplinary teaching to subject-oriented teaching. As shown in figure 1, the model is iterative. Once the concepts and skills are conceptually anchored in the respective subjects, they can evolve in a new interdisciplinary context, as part of a horizontal linkage [11].

In Denmark the notion of subject competences functions as a flexible framework for a description of what it means to master a subject independent of specific topics and specific levels. Competency is someone's insightful readiness to act appropriately in situations in a way that is guided by one's knowledge from a subject. Competence-based teaching permeates the Danish educational system, and there are fundamental potentials in terms of an overlap between

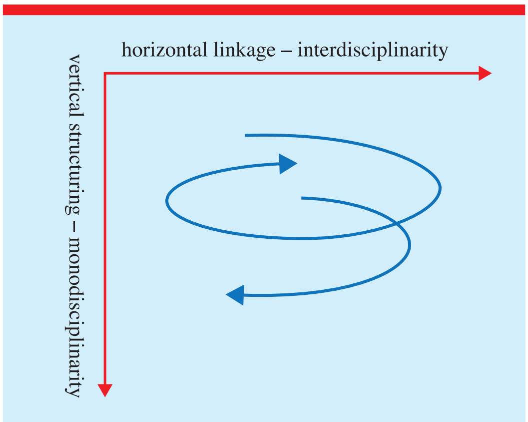

Figure 1. The spiral shape illustrates the repetitive movements between the horizontal and vertical phases.

subjects. Eight mathematical and four science competences are identified, and the competency of modeling is identified both as a mathematical and science competence. In mathematics the modeling competence includes structuring an intra- or extra-mathematical situation to be modelled, mathematizing the situation, analyzing and tackling the model, interpreting the results, validation of the model, communicating about the model, monitoring the modeling activity [12]. The reference to the modeling of extra-mathematical situations underscores that the competency should not be considered as a subject specific competency, but as an interdisciplinary competency [13]. Modeling is a specific problem solving strategy with scientific and pragmatic purposes, and as a rule scientific and everyday life problems call on modeling and do not accept traditional and historical determined boundaries between subjects. Thus the interdisciplinary competency of modeling is the putty that holds together the interacting subjects.

Student projects: interdisciplinary mathematics and physics teaching activities

The course 'Modeling and interdisciplinary teaching' is based on the didactical framework outlined above, a syllabus of research literature concerning interdisciplinarity, and activities involving the student-teachers in developing sequences of interdisciplinary teaching. For the implementation of the course the students are required to do a final exam project in groups of two to four students, where they design an imagined interdisciplinary teaching module and clearly (i) describe the form and extent of the interdisciplinarity, and the place and role of modeling, and (ii) argue for their design choices on the basis of the course literature.

In the following, four selected final exam projects are provided, in order to, on the one hand, give an idea of the kind of projects and prototypes for teaching activities that students devised, and, on the other hand, to illustrate the type of problems regarding interdisciplinary teaching that the students themselves came to realize as being imperative for the success of designing and implementing interdisciplinary teaching activities at upper secondary level. In the following, 'students' refers to the student-teachers enrolled at the course, while 'pupils' refers to the intended target group of the designed modules; upper secondary school pupils.

In the first example the students have chosen the theme breaking distance for a teaching module involving mathematics and physics. The motivation for the theme is that it addresses the core content of the basic level in the subjects of physics and mathematics, and the relevance to upper secondary school pupils, several of whom prepare to undergo a driving test. Besides functioning as a concrete example of application of differential and integral calculus in mathematics, and forces and kinematics in physics, the theme also provides the pupils with insight into the consequences of failing to comply with the traffic rules, and the challenges when driving on slippery surfaces. The teaching module starts by introducing the pupils to breaking distances and road safety through YouTube streams and short relevant articles that link breaking distances and road safety. This is followed by a series of tests aiming at experimentally investigating the relationship between initial speed and breaking distance. Related data sets of initial speed and breaking distance are collected, processed and evaluated on site. In the last part of the test series focus is on the effect of changes in the coefficient of friction, which can respond to slippery roads. In their concluding remarks the students point to the fact that the module forces the subjects to support each other and address a joint problem, namely construction of a model of the relationship between speed and breaking distance. Mathematics is applied in the analysis of the physical tests, and physics supports mathematics by forming empirical connection between the subjects.

In the second example the students outline a module entitled 'Energy and sustainability' incorporating physics, mathematics and social sciences. The rationale for choosing this theme is that it is a current issue in the public debate, plays an implicit role in the daily life of the pupils, and has great significance for their future life. Energy is part of the core content of physics at grade 10, and according to the students it could easily interact with the core content of mathematics and social sciences. The core idea of the module is to involve the pupils in developing a model of how energy production of a wind turbine depends on the wind speed, and then subsequently investigate the correspondence between wind speed and energy production experimentally with a model wind turbine. The pupils must perform regressions on data from the experiment to examine whether there may be a linear, exponential or power connection between energy production and wind speed. The climate is then addressed in a decision-making process context with focus on energy and climate policy. The students accentuate the theme of energy and sustainability as an example of how pupils with an interest in social sciences can be motivated by physics. By involving a societal perspective on problems in physics and applying knowledge from physics, mathematics, and social sciences, the relevance of physics is enhanced and a better basis for learning physics is created. Also the students emphasize the potential for developing interdisciplinary teaching if teachers from different subjects collaborate about teaching and learning, as members of a professional learning community, where expertise is seen as collective and based on knowledge that is shared and developed through collaboration.

In the third example the students start out with a critique of the traditional mono-disciplinary approach to the concept of exponential growth functions, which according to students does not have a cognitive value, as it does not address the transferability of the concept. Pointing out that exponential growth functions are both a central concept in mathematics and play an important role in the subjects of natural sciences as a modeling tool, the students suggest a module encompassing mathematics, chemistry, physics and biology.

Mathematics acts a pivotal point in the module and interdisciplinarity is addressed through three themes: (i) refining of raw sugar (mathematics and chemistry), (ii) radioactive decay (mathematics and physics), and (iii) pasteurization of milk (mathematics and biology). The teaching module starts with a warm-up activity, where the pupils use dice rolling to simulate a process, e.g. radioactive decay, which can be modelled by exponential growth functions. Then follows group work centered on solving contextual problem tasks rooted in two disciplines, where one is mathematics. The module is completed with a presentation of the outcome of the group work in plenary sessions. In the concluding remarks, the students note that teaching becomes more authentic through the inclusion of models and modeling. Furthermore, the students address the organizational challenges of interdisciplinary teaching and emphasize that teachers need knowledge about models and modeling to be able to organize interdisciplinary modeling activities in an appropriate manner.

The theme of the last example is movement with a special focus on jumping. The module encompasses mathematics, physics and physical education, and addresses the topics movement and sports studies, mechanics, dynamics, experimental work and vector calculus. The module is intended for subject packages including the subject of physical education. The students assume that pupils enrolled in such subject packages are keen to improve their physical performances. The core idea of the module is to involve the pupils actively in sports activities to optimize their physical performance, through better understanding of their body and application of the laws of physics and mathematical tools, like vector calculus. The module is centered on a forward somersault with run-up, which the pupils have to perform. The jumps are video recorded, and through the analysis of the video recordings of their own jumps, the pupils gain a better understanding of their body, feel the physical laws on their own body, and apply mathematics to predict the optimal body postures for a specific jump. In their concluding remarks the students address the relationships between the three involved disciplines. According to the students the interplay between physical education and physics is well expressed, whereas the role of mathematics is more indirect. Nevertheless, the students consider mathematics as a discipline on an equal basis with the other two, and emphasize that mathematics through the module will play the role of being an ancillary discipline as well as an integrating discipline, e.g. when vector calculus is applied for modeling a jump. Finally the students point out that planning interdisciplinary teaching is time-consuming and demands collaboration between the involved teachers. Furthermore teachers may be reluctant to familiarize themselves with other disciplines and ways of thinking.

Concluding remarks

Mathematics and physics have a close relationship, and this should be reflected in frequent joint instructional sequences between the two subjects. The examples of interdisciplinary modules from the students-teachers' exam projects show that the students are able to articulate a broad spectrum of interdisciplinary themes capturing the relationship between mathematics and physics. The didactical framework provides the students with a structure for identifying interdisciplinary themes with a significant content for the participating subjects, and modeling serves as the unifying activity in the students' modules. From both a teacher preparation course design perspective, as well as one of purely educational research, such observations lend credence to the idea that competency development can be seen as a lynchpin for interdisciplinary activities, not just across mathematics and natural sciences, but also across disciplines from all faculties.

It is worth noticing that several of the exam projects in addition to mathematics and physics include subjects like chemistry, biology, sports, and social sciences. This reflects the fact that some of the students have a background in other natural sciences, or in the humanities, or in social sciences. This led to a learning situation in which students were invited to negotiate the meaning of modeling in different subjects, and the students demonstrated the capacity to integrate two or more subjects in an interdisciplinary module.

In their exam reports, the students emphasize the relevance and motivational aspects of an interdisciplinary approach and modeling competence as the link between the interdisciplinary context and the subjects. In addition, the students are aware of the challenges of interdisciplinary teaching.

Designing an appropriate interdisciplinary experience is not an easy task for teachers, and the students point out that this kind of teaching is more demanding mono-disciplinary teaching. Teachers worry about losing subject identities, and teachers with a strong mono-disciplinary schooling may not see themselves capable of teaching in an interdisciplinary way. If the teachers are unfamiliar with common objectives and applications in mathematics and physics, students are led to believe that mathematics and physics are unrelated entities.

In order for future interdisciplinary teaching to be sustainably integrated in teachers' practice, more systemic changes need to occur at the level of the in-service teaching communities and in terms of the continued professional development that occurs with teachers at a school.

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